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## OBITUARY

Graham Everest, 1957–2010



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“He heals the brokenhearted and binds up their wounds. He determines the number of the stars and calls them each by name” Psalm 147, v. 3–4 (NIV).

1. *Introduction*

Graham Everest was an inspirational mathematician, who touched the mathematical lives of a great many more than the thirty with whom he published. He collaborated widely, and those who worked with him enjoyed the joy he took in learning and doing mathematics. His creative leaps could generate mistakes, but these were often of the fruitful variety, triggering new ideas and ways forward. While his primary research area was in algebraic number theory, he was always interested in interactions between different parts of mathematics, and his research publications touched on aspects of analytic and computational number theory, dynamical systems, and logic. He was an enthusiastic user of computational methods to inform theoretical thinking and to enhance his approach to teaching. Alongside conventional research publications, the exposition of mathematics mattered greatly to him. While it sadly came after his death, the fact that one of his publications [81] won the 2012 Lester Ford Award from the Mathematical Association of America would have brought him huge pleasure. He was a dedicated and enthusiastic teacher, and deservedly won a UEA Excellence in Teaching Award from student nominations in 2005.

2. *Personal life*

As a person Graham was warm, generous, and possessed of a large hinterland. This ranged from playing bridge, amateur dramatics, interests in world music, the writings of Carl Jung, religious mysticism and cake-baking, to arranging wine and poetry evenings. His Christianity eventually saw him move away from Evangelical free churches and an — at times controversial

— advocacy of young earth creationism [55], towards the Church of England and a humble and questioning approach to all such matters. Eventually Graham's faith led to his undergoing theological training, initially as a lay reader, and later to his being ordained as a non-Stipendiary Minister in 2006 at a ceremony in Norwich Cathedral. Parishioners from Colney and Cringleford in Norwich appreciated the warmth, wit, and love he brought to his vocation, in which he felt a particular calling to the ministry of encouragement.

The year 2008 brought enormous difficulties to Graham's life. At the start of the year he received the devastating diagnosis of an aggressive and inoperable form of cancer. Later his own at times complicated personal struggles led to him leaving his role in the Church. He faced death much as he had faced life, with honesty, candour, wit, and passion. Shortly before he passed away at home, he was reconciled with the Church and reinstated as a priest.

Graham remained mathematically active throughout his illness, and continued to teach while he was physically able to. One of his last publications [81] was dedicated to 'Mulbarton Ward and the Weybourne Day Unit in Norwich', where he had received medical and hospice care respectively.

Graham is survived by his children James, Philip, and Rebekah, and his wife Sue.

### 3. *Mathematical career*

Graham was born into a working-class family where University was not seen as an automatic path to follow, in Southwick, West Sussex. His early childhood was happy, and his playful personality developed through inventive games played with his older sister Jan. Primary school was not an easy time for Graham, perhaps as a result of boredom. His mathematical talent was spotted at High School, and Graham was the first person at his school to take an S-level examination in Mathematics. More interested in playing bridge than in his formal studies at school, he dropped an A-level in order to devote more time to this. He followed his father's advice, and started an auditing apprenticeship – but this failed to fulfil him intellectually. Happily, Graham's Mathematics teacher helped him apply to study Mathematics in the next academic year.

Bedford College (now part of Royal Holloway, University of London), where Graham started his degree in 1977, expanded Graham's horizons in many ways. He embraced literature, art, and music, starting several life-long interests. One of several great friends he made, who later went on to become a monk, shared his Christian faith with Graham, and the outworking of this faith was to influence all aspects of Graham's later life, and to have a profound influence on members of his family and many of his friends. Graham's mathematical powers were clear, and he won both first and second year prizes, later explaining to his wife Sue that the missing third year prize had not been won because it did not exist. He went on to study for a doctorate at King's College London under the supervision of Colin Bushnell, and during this period was both influenced and supported by Martin Taylor and Albrecht Fröhlich. In the Summer of 1983 he completed his PhD thesis [2] and married Sue, who had by then also graduated from Bedford College. Graham turned down a temporary lectureship at Sheffield (who were kind enough to release him) in order to take up a permanent lectureship at UEA.

During his long career at UEA, Graham was always energetically dedicated to both education and research. For some years in the 1980s the flame of pure mathematics research at UEA was largely kept alive by the efforts of Graham and Alan Camina. Graham took great pleasure in the steady growth in the research power of the UEA Mathematics Department that took place thereafter, and was openly delighted by the vibrant atmosphere that emerged.

#### 4. Mathematical research

Graham’s research outputs appeared in the form of some 70 research and research-expository papers and three monographs [43], [59], and [62]. While always remaining number-theoretical, his work may be roughly categorized into three main areas: Diophantine analysis, dynamical systems, and recurrence sequences.

##### 4.1. Diophantine analysis and counting problems

Graham’s early interests lay in using analytic and Diophantine methods to understand counting and distribution problems. In his first papers he studied a conjecture of Bushnell ⟨4⟩, using Schmidt’s subspace theorem to study the distribution of normal integral generators in number fields [1], [3]. He saw how one could employ Alan Baker’s extremely powerful work on Diophantine approximation ⟨2⟩ in the investigation of quite complex algebraic constraints. This particular combination was unprecedented at the time, and already exhibited the originality and insight of Graham’s later work. He went on to use analytic methods in the same area [5], setting the tone for much of his later work: bringing deep Diophantine results and an analytic toolbox to bear on counting or growth problems.

Motivated by the work of Evertse ⟨7⟩ on the  $S$ -unit theorem, Graham used analytic methods of the Hardy–Littlewood type to study the distribution of  $N(\mathbf{x}) = \prod_v |c_0x_0 + \dots + c_nx_n|_v$  for  $x_i$  running through  $S$ -units in a number field [13]. He also studied norm-form equations from a similar perspective [11], [22], [38].

##### 4.2. Dynamical systems of algebraic origin

I first met Graham across an interview table in 1991. Despite the constraints of the setting, it was quickly apparent that we shared an interest in the remarkable, then recent, work of Schlickewei and others on the subspace theorem, and the resulting insights into the solution of  $S$ -unit equations in fields of characteristic zero ⟨8⟩, ⟨14⟩, ⟨15⟩. In algebraic dynamics this had led directly to the result that a mixing action of  $\mathbb{Z}^d$  by automorphisms of a compact connected group is mixing of all orders ⟨16⟩, and for Graham this unexpected instance of number-theoretical results finding applications in dynamical systems was fascinating. This began a conversation between us that lasted almost twenty years, each bringing in tools and techniques from our own areas of mathematics.

Algebraic dynamics is also a setting in which the Mahler measure

$$m(f) = \int_0^1 \dots \int_0^1 \log |f(e^{2\pi i s_1}, \dots, e^{2\pi i s_d})| ds_1 \dots ds_d$$

of a polynomial  $f \in \mathbb{Z}[u_1^{\pm 1}, \dots, u_d^{\pm 1}]$  arises naturally as the topological entropy of a  $d$ -dimensional dynamical system ⟨11⟩ associated to the polynomial  $f$ . In the case  $d = 1$  the most natural proof exploits properties of the adèle ring  $\mathbb{Q}_{\mathbb{A}}$ , and in the simplest case of linear polynomials the notion of Diophantine height appears in a particularly transparent way ⟨12⟩. If  $d = 1$  and  $f(u_1) = bu_1 - a$  then the associated dynamical system is given by the map  $\alpha$  dual to  $x \mapsto \frac{a}{b}x$  on the character group of  $\mathbb{Z}[\frac{1}{b}]$ , and the topological entropy is given by

$$h(\alpha) = \sum_{p \leq \infty} \log \max\{0, |\frac{a}{b}|_p\} = \log \max\{|a|, |b|\} = m(f).$$

Once again Graham was intrigued to find something he had studied in a number-theoretical setting making an appearance in dynamical systems. Two aspects of Mahler measure became of particular interest to Graham — its special values, which later came to have interpretations in terms of periods of mixed motives ⟨6⟩, and the classical problem of Lehmer ⟨10⟩ asking if

$$\inf\{m(f) \mid m(f) > 0\} > 0.$$

This rich circle of questions, a mixture of Diophantine analysis and arithmetic geometry, triggered a long interest in the interplay between dynamical systems and arithmetic, leading to several research projects and the monograph [43] on the relationship between heights of algebraic numbers and topological entropy in algebraic dynamics.

One theme that Graham pursued in this area concerned growth in periodic orbits for automorphisms of solenoids, which relates directly to questions about the arithmetic of terms of recurrence sequences and linked to his long-standing interest in the arithmetic of linear and bi-linear recurrence sequences. These questions are of interest in algebraic dynamics because they arise in any attempt to understand typical or generic compact group automorphisms. The simplest examples boil down to questions of the following shape. What can be said about growth in expressions like

$$(2^n - 1) \prod_{p \in S} |2^n - 1|_p$$

as  $n \rightarrow \infty$  for various subsets  $S$  of the set of all primes  $\mathbb{P}$ ? Trying to develop a good understanding of what can be said here, and how much averaging is needed to smooth out the Mersenne-like quirks in the prime factorization of linear recurrence sequences, held Graham's attention on and off for several years, and eventually led to a rather complete understanding of the two extreme cases  $|S| < \infty$  in [70] and  $|\mathbb{P} \setminus S| < \infty$  in [78]. The latter case brought Graham particular pleasure as it involved Dirichlet series and analytic issues relating to an asymptotic counting problem in a novel setting, often with such poor analytic behaviour on the critical line that Tauberian theorems could not be applied. The large middle ground — the uncountable collection of infinite sets of primes with infinite complement — saw some early partial results [36] leading eventually to constructions of families of examples exhibiting continua of different orbit growth rates for a suitably averaged measure of orbit growth  $\langle \mathbf{1} \rangle$ .

#### 4.3. Recurrence sequences, heights, and logic

No mathematician is unaware of open problems concerned with the appearance of primes in the sequence  $(2^n - 1)$  ('Mersenne' primes); only slightly less well-known is the well-understood appearance of primitive divisors in the same sequence (a primitive divisor of a term is a prime divisor that does not divide any earlier term). Many mathematicians might find the briar patch of notorious difficulties surrounding Mersenne primes, or the over-manicured and well-trodden garden surrounding the question of primitive divisors in sequences like  $(a^n - b^n)$ , with its frequently re-proved and definitive Zsigmondy–Bang theorem  $\langle \mathbf{19} \rangle$ , less than appealing. It was typical of Graham that in both cases he saw rich territory for new exploration and extension, and he was particularly enthused by the way in which Bilu, Hanrot and Voutier  $\langle \mathbf{3} \rangle$  were able to use modern Diophantine results to solve a century-old problem, proving that the  $n$ th term of any Lucas or Lehmer sequence has a primitive divisor for  $n > 30$ . This paper, with its sophisticated use of deep Diophantine results and technical skill to produce a result that was startling both in its uniformity and in its highly effective bounds, was a great inspiration to Graham.

Graham studied the arithmetic of linear recurrence sequences from many different perspectives, and some of these questions were pursued by doctoral students under his supervision. The Zsigmondy–Bang theorem involves some key estimates requiring an understanding of the rate of growth in a linear recurrence sequence, and the delicate issues arising here may already be seen in Lehmer–Pierce sequences  $(\Delta_n(f))$ , where  $f(x) = \prod_{i=1}^d (x - \alpha_i)$  is a monic integer polynomial with no cyclotomic factor, and  $\Delta_n(f) = \prod_{i=1}^d |\alpha_i^n - 1|$ . If no  $\alpha_i$  has unit modulus, then the growth rate is clear, while if  $f$  has zeros of unit modulus then a *deus ex machina* like

Baker's theorem is required to know that

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \Delta_n = \sum_{|\alpha_i| > 1} \log |\alpha_i| = m(f),$$

the logarithmic Mahler measure or height of  $f$ . The question of prime appearance remains inaccessible for linear recurrence sequences, triggering one of Graham's excursions into computational work. Writing  $(n_j)$  for the sequence of indices for which  $\Delta_{n_j}$  is prime, he used Baker's theorem to show that  $(n_j)$  has only finitely many composite terms [49], and built on Wagstaff's heuristics to argue that  $j/\log \log \Delta_{n_j}$  should converge as  $j \rightarrow \infty$  to a quantity controlled by  $m(f)$ , adding further numerical evidence to the original insight of Lehmer [10]. Some of these ideas found more rigorous outlet in work of his students, notably that of Flatters [9], where uniform bounds for the appearance of primitive divisors in sequences associated to real quadratic units are found.

Graham was also interested in elliptic or bi-linear recurrence sequences. For an elliptic curve  $E$  defined over  $\mathbb{Q}$  and given in Weierstrass form, and a non-torsion point  $P \in E(\mathbb{Q})$ , there is an associated integer sequence  $(B_n)$  defined by  $[n]P = (\frac{A_n}{B_n^2}, \frac{C_n}{B_n^3})$ . Here the work of Silverman [17] showing that all but finitely many terms of an elliptic divisibility sequence have a primitive divisor proved to be inspirational. Graham formulated a conjectural view of the arithmetic properties of these sequences, including the striking suggestion that the number of prime terms in  $(B_n/B_1)$  should be bounded uniformly. He was able to establish many special cases, including strong results conditional on Lang's height conjecture [71], [61]; generalizations to Somos sequences associated to sequences like  $([n]P + Q)$  in [64]; generalizations to Siegel and Hall theorems [74]; and results on primitive divisors [75], including highly effective bounds across certain families of curves [67].

In both the linear and the elliptic theory, a critical role is played by notions of height. In the linear case, this may be expressed as a Mahler measure, and in the elliptic case as a Neron–Tate height. Pursuing a broad thematic analogy between the two theories, Graham introduced in [35] an elliptic Mahler measure  $m_E(F)$  as a sum of local integrals, using an interesting integral representation for the canonical local height on  $E$ . In particular, this gives rise to a beautiful elliptic analogue of Jensen's theorem, and the result that  $m_E(F) = 0$  if and only if the roots of  $F$  are the  $x$ -coordinates of division points (a direct elliptic analogue of Kronecker's lemma in the classical case).

The possibility of relating Hilbert's 10th problem for  $\mathbb{Q}$  to arithmetic properties of elliptic divisibility sequences (see Poonen [13]) fascinated Graham, leading to an elegant argument [73] building on Poonen's work to find a partition  $S \sqcup T$  of the primes into recursive sets with the property that Hilbert's 10th problem is undecidable for both  $\mathbb{Z}_S$  and  $\mathbb{Z}_T$  (later generalized to number fields in [80]). How Graham came to be interested in these questions highlights several aspects of the way he worked. Gunther Cornelissen, following a talk by Thanases Pheidas in Gent in 1999 raising the possibility of attacking the problem using arithmetic properties of elliptic divisibility sequences, started exploring some of the literature on these sequences, leading to the results in [5]. For quite different reasons, Graham had been developing a web site collecting material and his own thoughts on elliptic divisibility sequences, bringing together quite disparate strands of thoughts growing from Morgan Ward's classical treatment [18]. Gunther came across this web site while researching the topic. This led to Graham attending a mini-workshop in Oberwolfach on Hilbert's Tenth Problem, and to collaboration with some of the participants, including Alexandra Shlapentokh [80] and Kirsten Eisenträger [73], [80]. Using the web to find another mathematical environment with new concepts to learn, people to meet, and friends to make, where his knowledge of elliptic divisibility sequences could be applied, brought him great pleasure in the last few years of his life.

The impact Graham had as a person on several different mathematical communities was reflected in two international conferences. The first — ‘Diverse faces of arithmetic’ — which he much enjoyed, was a retirement conference at UEA in December 2009, bringing together people from number theory, dynamical systems, integrable systems, and logic. The second, ‘Definability in Number Theory’ at the University of Gent in September 2010, was dedicated to his memory and brought together mathematicians interested in the question of which sets and structures can be defined or interpreted in the existential or first-order theory of rings and fields.

### 5. *Research students*

Graham was a dedicated and thoughtful supervisor. The breadth of his interests and his determination to continue to supervise graduate students as his illness progressed meant that joint supervisions arose naturally, and both Shaun Stevens and I enjoyed several joint supervisory experiences with him.

#### RESEARCH STUDENTS SUPERVISED OR JOINTLY SUPERVISED BY GRAHAM EVEREST

Alice Miller, PhD 1988, ‘*Effective subspace theorems for function fields*’

Bríd Ní Fhlathúin, PhD 1995, ‘*Mahler’s measure on Abelian varieties*’

Peter Panayi, PhD 1995, ‘*Computation of Leopoldt’s  $p$ -adic regulator*’

Vijay Chothi, PhD 1996 (with Tom Ward), ‘*Periodic points in  $S$ -integer dynamical systems*’

Paola D’Ambros, PhD 2000 ‘*Algebraic dynamics in positive characteristic*’

Christian Röttger, PhD 2000, ‘*Counting problems in algebraic number theory*’

Peter Rogers, MPhil 2003, ‘*Computational aspects of elliptic curves*’

Patrick Moss, PhD 2003, ‘*The arithmetic of realizable sequences*’

Victoria Stangoe, PhD 2004 (with Tom Ward), ‘*Orbit counting far from hyperbolicity*’

Helen King, PhD 2005 (with Tom Ward), ‘*Prime appearance in elliptic divisibility sequences*’

Jonathan Reynolds, PhD 2008 (with Shaun Stevens), ‘*Extending Siegel’s theorem for elliptic curves*’

Ouamporn Phuksuwan, PhD 2009 (with Shaun Stevens), ‘*The uniform primality conjecture for the twisted Fermat cubic*’

Anthony Flatters, PhD 2010 (with Tom Ward), ‘*Arithmetic properties of recurrence sequences*’

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